Ramsey taxation in the sequence space

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Optimal Policy with heterogeneous agents?

- So far, focused on HANK, discussed lots of **positive** questions
  - e.g. effects of fiscal policy on output, monetary policy, …
- Very little work on **normative** implications (hard!)
  - optimal capital & labor taxation? optimal level of public debt?
- **Next:** A first step …
  - Optimal long-run fiscal policy
  - … in a canonical HA model without NK

Ramsey steady state

- We focus on characterizing the **Ramsey steady state (RSS)**
  - long-run steady state of the full-commitment Ramsey plan
- A long literature characterizes the RSS in simpler models (RA, TA)
  - e.g. Chamley (1986), Judd (1985), Straub Werning (2020)
- We study the RSS in **neoclassical HA models**, à la Aiyagari

What has been done on this

- **Not much!** Aiyagari (1995) and Chien Wen (2023) at
- Dyrda Pedroni (2022): focus on transition (not RSS)
- Acikgöz et al (2022): complex system of FOCs to co

$$\mathcal{L} = \int \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big\{ \left(u(c_t,n_t) + u_c(c_t,n_t)\left((\eta_t(a_t-\underline{a})-\theta_t)(1+\bar{r}_t) - (\eta_{t+1}(a_{t+1}-\underline{a})-\theta_{t+1})\right)\right) + \gamma_t\Big(F(K_t,N_t) - \delta K_t + B_{t+1} - G_t - T_t - (1+\bar{r}_t)B_t - \bar{r}_t K_t - \bar{w}_t N_t\Big)\Big\} d\mu_0. \quad (12)$$

To simplify the notation, we define $\lambda_{t+1} = \eta_{t+1}(a_{t+1}+\underline{a}) - \theta_{t+1}$. We derive FOCs from the Lagrangian in Appendix I and show that the interior solution of the Ramsey problem satisfies the following conditions:

$$\lambda_{t+1}: \quad u_c(c_t,n_t) = \beta(1+\bar{r}_{t+1})\mathbb{E}_t[u_c(c_{t+1},n_{t+1})] \text{ if } a_{t+1} > -\underline{a}, \text{ otherwise } a_{t+1} = -\underline{a}, \quad (13)$$

$$a_{t+1}: \quad u_c(c_t,n_t) + u_{cc}(c_t,n_t)(\lambda_t(1+\bar{r}_t) - \lambda_{t+1}) = \beta(1+\bar{r}_{t+1})\mathbb{E}_t[u_c(c_{t+1},n_{t+1}) + u_{cc}(c_{t+1},n_{t+1})(\lambda_{t+1}(1+\bar{r}_{t+1}) - \lambda_{t+2})] + \beta\gamma_{t+1}(F_K(K_{t+1},N_{t+1}) - \delta - \bar{r}_{t+1}) \text{ if } a_{t+1} > -\underline{a}, \text{ otherwise } \lambda_{t+1} = 0, \quad (14)$$

$$B_{t+1}: \quad \gamma_t = \beta(1+F_K(K_{t+1},N_{t+1}) - \delta)\gamma_{t+1}, \quad (15)$$

$$\bar{r}_t: \quad \gamma_t A_t = \mathbb{E}_t[u_c(c_t,n_t)\lambda_t + a_t(u_c(c_t,n_t) + u_{cc}(c_t,n_t)(\lambda_t(1+\bar{r}_t) - \lambda_{t+1}))], \quad (16)$$

$$\bar{w}_t: \quad \gamma_t N_t = \gamma_t(F_N(K_t,N_t) - \bar{w}_t)\frac{\partial N_t}{\partial \bar{w}_t} + \mathbb{E}_t\left[e_t n_t u_c(c_t,n_t) + \left(\frac{\partial c_t}{\partial \bar{w}_t}u_{cc}(c_t,n_t) + \frac{\partial n_t}{\partial \bar{w}_t}u_{cn}(c_t,n_t)\right)(\lambda_t(1+\bar{r}_t) - \lambda_{t+1})\right]. \quad (17)$$

Large literature computes "optimal steady state" (OSS) instead of RSS

- issue: OSS assumes infinitely patient planner, ignores transitional dynamics

[e.g. Aiyagari McGrattan 1998 …]

Next: New “sequence-space” approach

1. Heterogeneous-agent household side, introduce **discounted elasticities**
2. Set up Ramsey problem and derive FOCs
3. Numerically evaluate FOCs, get Ramsey steady state for many specifications

Note: Generalizes to other <u>stationary</u> household sides (bonds in utility, OLG,…)

1. Heterogeneous-agent household side

Households

Just like before, except hours are optimally chosen by households:

$$\max_{\{c_{it}, n_{it}, a_{it}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_{it}, n_{it})$$

$$c_{it} + a_{it} = (1 + r_t) a_{it-1} + (1 - \tau_t) e_{it} n_{it} \qquad a_{it} \geq 0$$

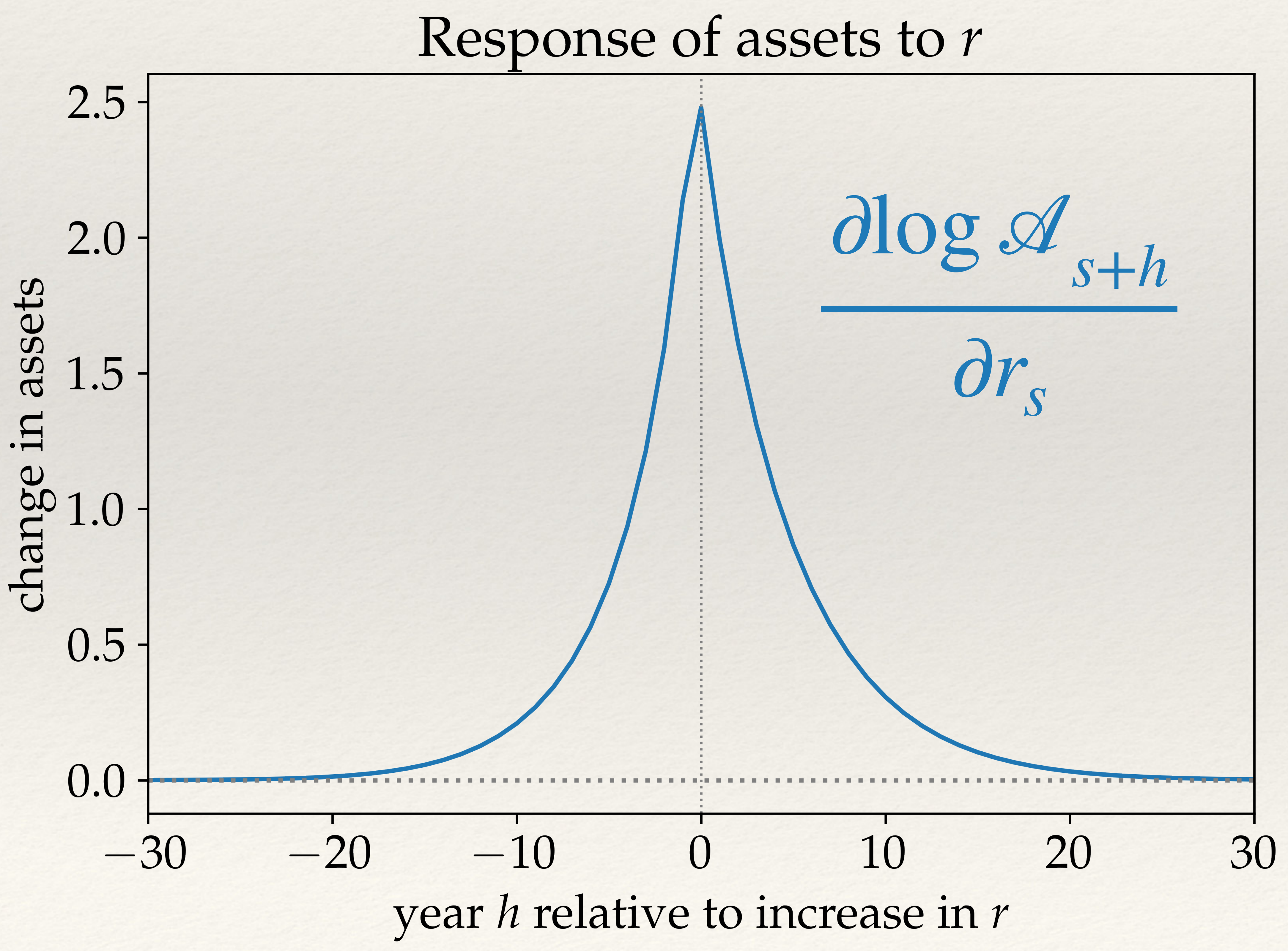
Inputs: interest rate and labor tax

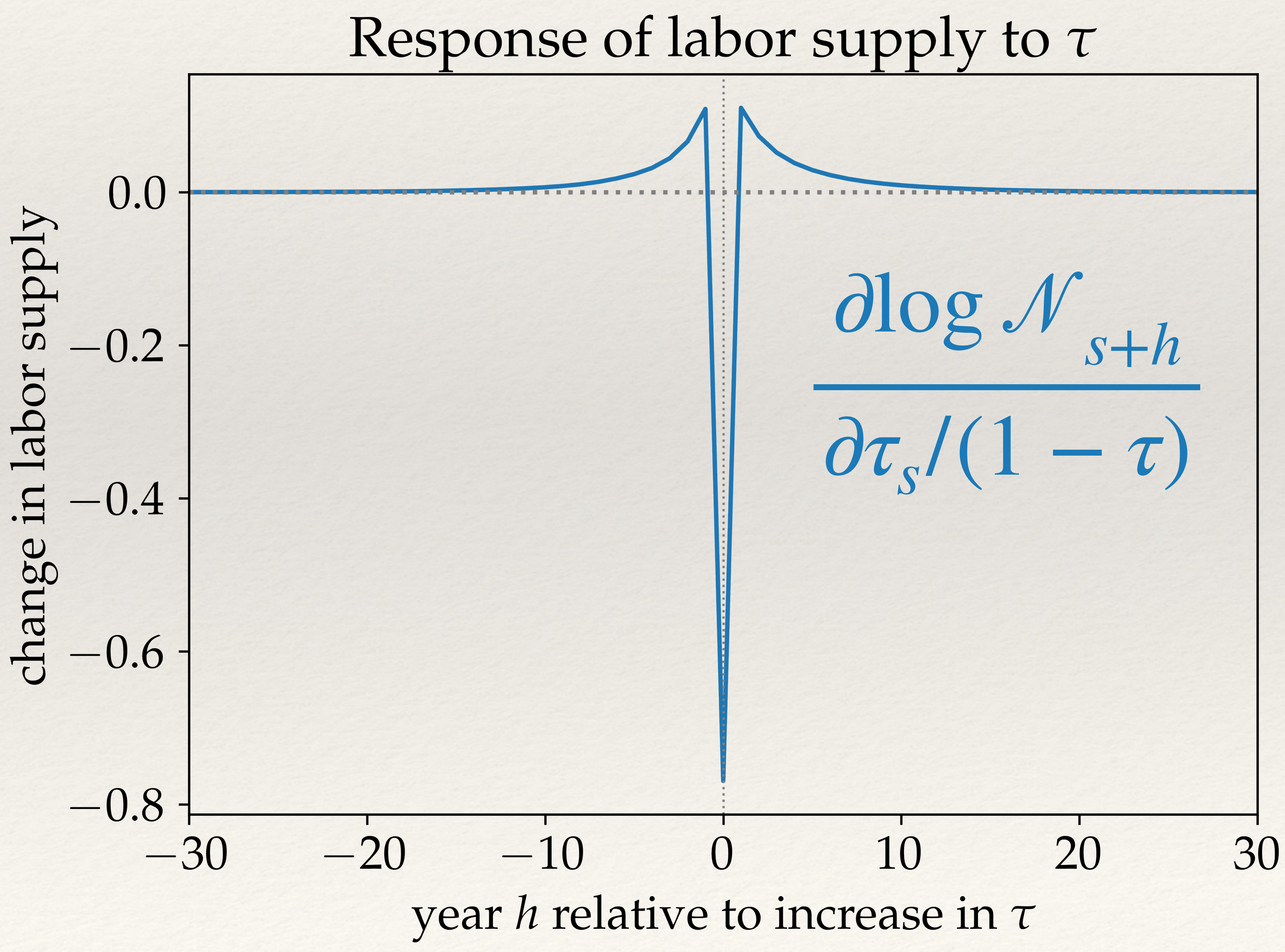
Given $\{r_t\}$, $\{\tau_t\}$, can again aggregate household behavior using **sequence-space functions**:

Assets	$\mathscr{A}_t(\{r_s, \tau_s\}) = \int a_t dD_t$
Effective labor	$\mathscr{N}_t(\{r_s, \tau_s\}) = \int en_t dD_t$
Utility	$\mathscr{U}_t(\{r_s, \tau_s\}) = \int u(c_t, n_t) dD_t$

Infinitely anticipated shocks

- Consider **anticipated one-time** shock at some far-out future date s





δ-discounted elasticities

- Define "discounted" version of these derivatives (around steady state with r, τ)

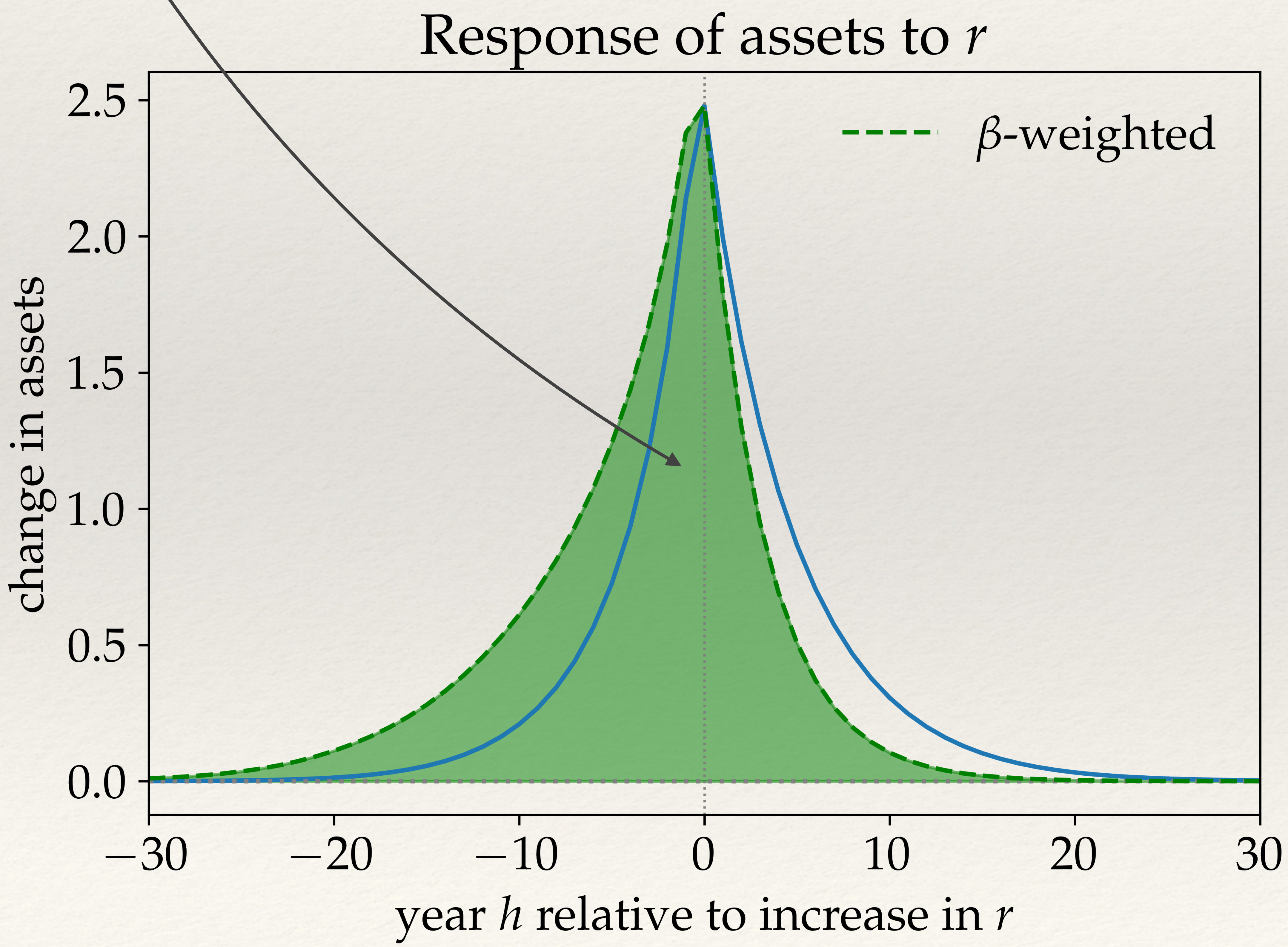
$$\epsilon^{A,r}(r,\tau) \equiv \lim_{s\to\infty} \sum_{h=-\infty}^{\infty} \delta^h \frac{\partial \log \mathcal{A}_{s+h}}{\partial r_s} \qquad \epsilon^{N,\tau}(r,\tau) \equiv \lim_{s\to\infty} \sum_{h=-\infty}^{\infty} \delta^h \frac{\partial \log \mathcal{N}_{s+h}}{\partial \tau_s/(1-\tau_s)}$$

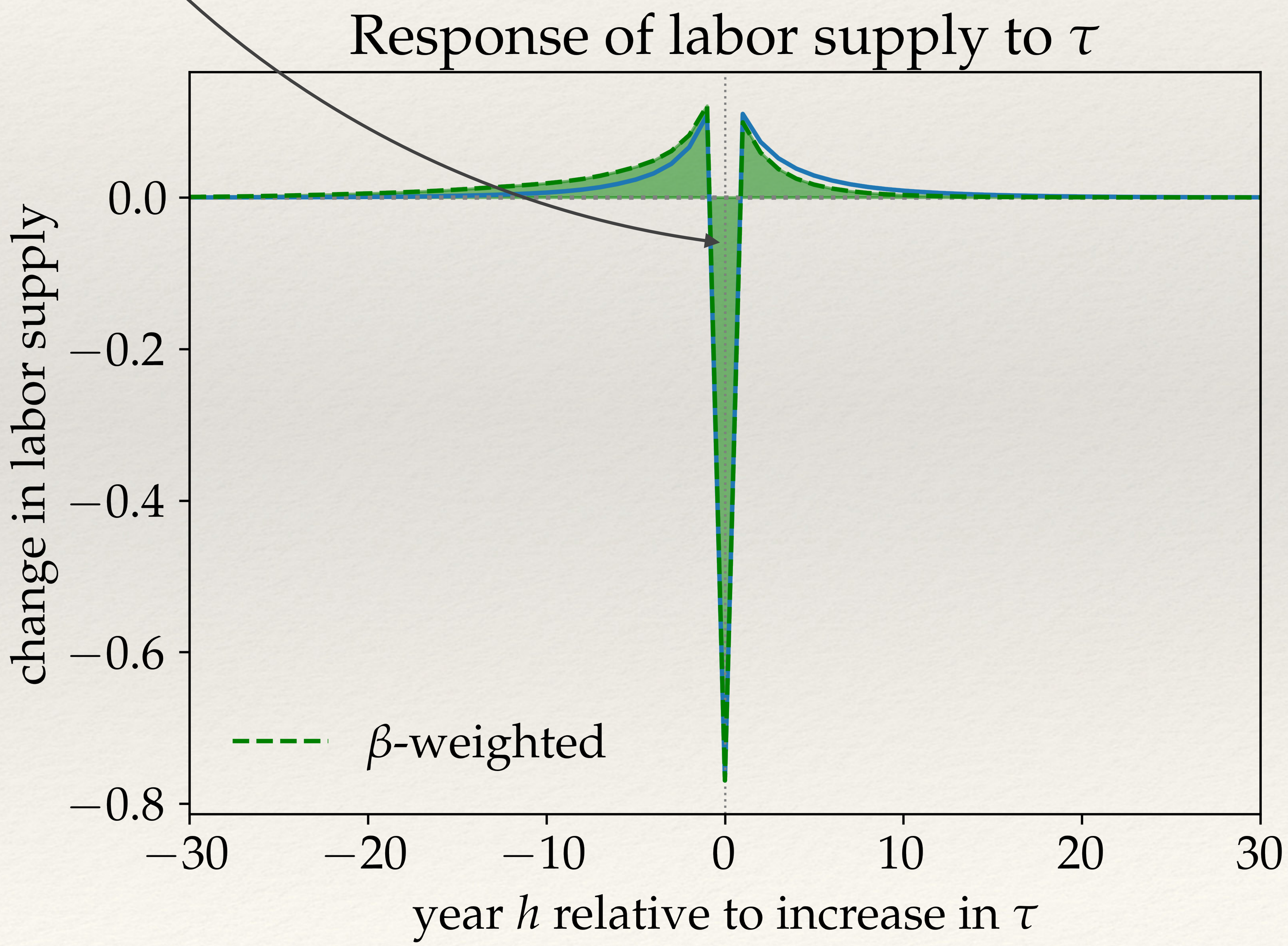
- These elasticities are discounted with some δ (later social discount factor)
- Well-defined for $\delta \in [\beta,1]$ precisely because the model is stationary!
- Define all the other elasticities similarly, e.g. $\epsilon^{N,r}$, $\epsilon^{A,\tau}$, $\epsilon^{U,r}$ etc

β-discounted elasticities

$$\epsilon^{A,r} \equiv \lim_{s\to\infty} \sum_{h=-\infty}^{\infty} \beta^h \frac{\partial \log \mathcal{A}_{s+h}}{\partial r_s} \approx 25$$

$$\epsilon^{N,\tau} \equiv \lim_{s\to\infty} \sum_{h=-\infty}^{\infty} \beta^h \frac{\partial \log \mathcal{N}_{s+h}}{\partial \tau_s/(1-\tau)} \approx 0.15$$





2. Dual Ramsey problem

Model description

- We've seen how we can summarize household behavior using "sequence space" functions $\mathscr{A}_t, \mathscr{N}_t, \mathscr{U}_t$

- **Next:**
  - set up the rest of the model: supply side, government policies
  - derive an implementability condition
  - set up the Ramsey problem!

Production and government policy

- Representative firm: $Y_t = \mathcal{N}_t$, pre-tax wage = 1
- Government: spends fixed $G > 0$ (can relax)
  - controls <u>labor taxes</u> $\{\tau_s\}$, budget constraint: $G + (1 + r_t) B_{t-1} = B_t + \tau_t N_t$

Implementability condition: $\{r_s\}$, $\{\tau_s\}$ part of an equilibrium iff

$$G + (1 + r_t) \mathcal{A}_{t-1}\left(\{r_s, \tau_s\}\right) = \mathcal{A}_t\left(\{r_s, \tau_s\}\right) + \tau_t \mathcal{N}_t\left(\{r_s, \tau_s\}\right)$$

Ramsey problem

Full-commitment Ramsey problem, with arbitrary social discount factor δ

$$\max_{\{r_s,\tau_s\}_{s=0}^{\infty}} \sum_{t=0}^{\infty} \delta^t \mathcal{U}_t(\{r_s, \tau_s\})$$

$$G + (1 + r_t)\, \mathcal{A}_{t-1}\left(\{r_s, \tau_s\}\right) = \mathcal{A}_t\left(\{r_s, \tau_s\}\right) + \tau_t \mathcal{N}_t\left(\{r_s, \tau_s\}\right)$$

- If solution converges to well-defined steady state $(r_s \to r < 1/\beta - 1, \quad \tau_s \to \tau < 1)$ we call this steady state a **Ramsey steady state (RSS).**
- Multiplier on the constraint λ_t may or may not converge!
  - For today, assume it does, $\lambda_t \to \lambda$. Relax this in the paper.

Characterizing the Ramsey steady state

$$\max_{\{r_s,\tau_s\}_{s=0}^{\infty}} \sum_{t=0}^{\infty} \delta^t \mathcal{U}_t(\{r_s, \tau_s\}) \qquad G + (1 + r_t)\, \mathcal{A}_{t-1}\left(\{r_s, \tau_s\}\right) = \mathcal{A}_t\left(\{r_s, \tau_s\}\right) + \tau_t \mathcal{N}_t\left(\{r_s, \tau_s\}\right)$$

- Begin with the FOCs with respect to r_s:

$$\underbrace{\sum_{h=-s}^{\infty} \delta^h \frac{\partial \mathcal{U}_{s+h}}{\partial r_s}}_{\to\ \epsilon^{U,r}\ \text{as } s \to \infty} + \underbrace{\sum_{h=-s}^{\infty} \delta^h \lambda_{s+h}\left(\frac{\partial \mathcal{A}_{s+h}}{\partial r_s}\right.}_{A\lambda \cdot \epsilon^{A,r}} + \underbrace{\tau_t \frac{\partial \mathcal{N}_{s+h}}{\partial r_s}}_{\tau N \lambda \cdot \epsilon^{N,r}} - \underbrace{(1 + r_t) \frac{\partial \mathcal{A}_{s+h-1}}{\partial r_s}}_{A\lambda(1+r)\delta \cdot \epsilon^{A,r}} \left.\vphantom{\frac{\partial}{\partial}}\right) - \underbrace{\lambda_s \mathcal{A}_{s-1}}_{\lambda A} = 0$$

Characterizing the Ramsey steady state

$$\max_{\{r_s,\tau_s\}_{s=0}^{\infty}} \sum_{t=0}^{\infty} \delta^t \mathscr{U}_t(\{r_s, \tau_s\}) \qquad G + (1 + r_t)\,\mathscr{A}_{t-1}\left(\{r_s, \tau_s\}\right) = \mathscr{A}_t\left(\{r_s, \tau_s\}\right) + \tau_t \mathscr{N}_t\left(\{r_s, \tau_s\}\right)$$

- From the r_s derivative around the (unknown) RSS:

$$\lambda^{-1}\epsilon^{U,r} = A - \big(1 - \delta\,(1 + r)\big)\,A\epsilon^{A,r} - \tau N\epsilon^{N,r}$$

- Same procedure applied to the τ_s derivative:

$$\lambda^{-1}\epsilon^{U,\tau} = (1 - \tau)\,N - \big(1 - \delta\,(1 + r)\big)\,A\epsilon^{A,\tau} - \tau N\epsilon^{N,\tau}$$

Two helpful objects: $\ell \equiv \dfrac{A}{(1-\tau)N}$ as *liquidity*; $m \equiv -\,\epsilon^{U,\tau}/\epsilon^{U,r} > 0$ as *effective MRS*.

RSS optimality condition

- If allocation converges to a well-defined RSS with interest rate r and tax rate τ, and if λ_t converges, then (r, τ) are characterized by:

1. The steady-state government budget constraint

$$G + r\mathscr{A}(r,\tau) = \tau\mathscr{N}(r,\tau)$$

2. Optimality condition

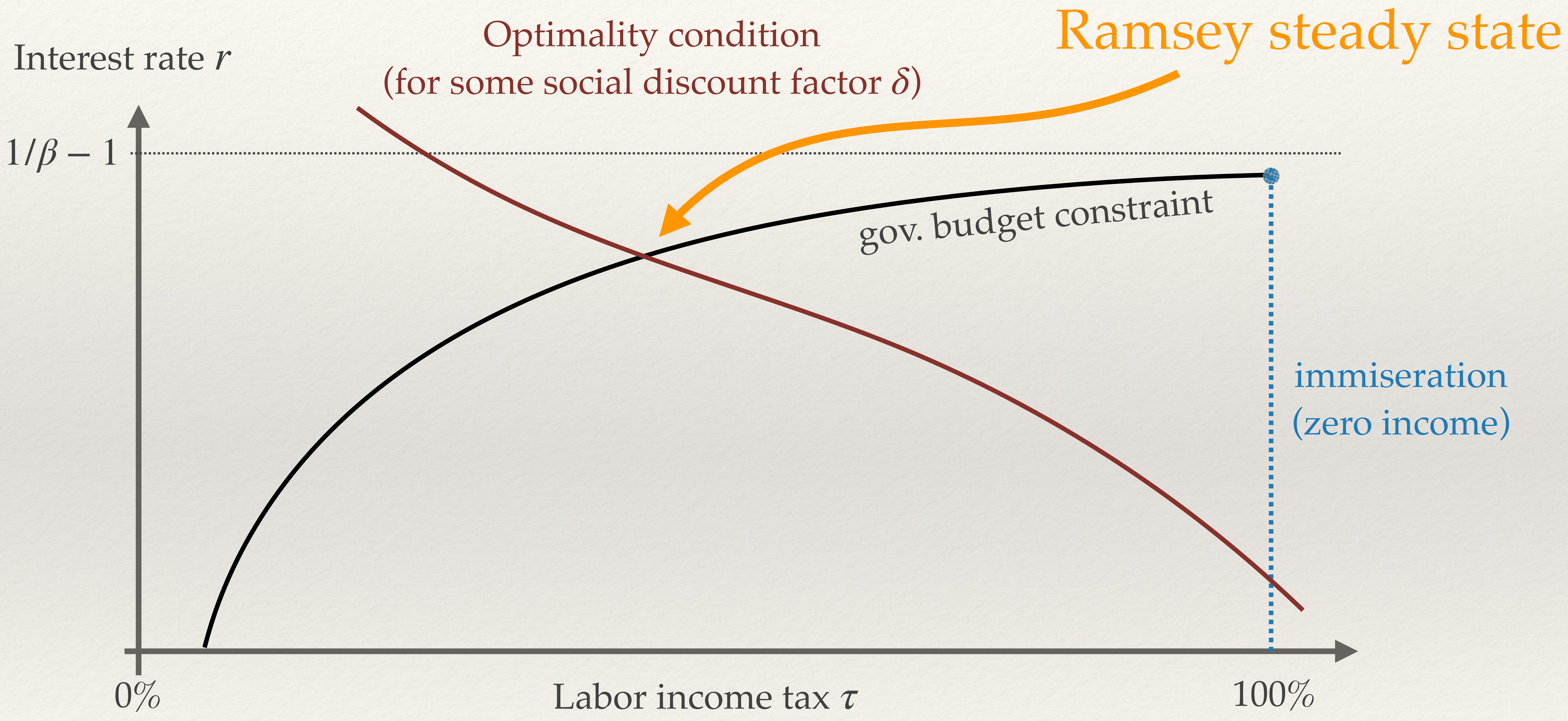
$$\underbrace{\left(1-(1+r)\delta\right)\ell\left(m\epsilon^{A,r}+\epsilon^{A,\tau}\right)}_{\text{liquidity benefit of greater debt}} - \underbrace{\frac{\tau}{1-\tau}\left(-\epsilon^{N,\tau}-m\epsilon^{N,r}\right)}_{\text{cost (?) lower labor supply}} - \underbrace{\left(\ell m - 1\right)}_{\text{cost: redistribution from workers to savers}} = 0$$

liquidity **benefit** of greater debt

cost (?) lower labor supply

cost: redistribution from workers to savers

The RSS first order condition



3. Searching for an RSS

Utility functions

- To solve this system of equations, need to go to the computer.
- Begin with $u(c, n) = \log c - v(n)$ with constant Frisch elasticity = 1
- Standard calibration: (details are not important)
  - AR(1) income process, initial debt = 100%, $G = 20\%$, initial $r = 2\,\%$
- **Idea:** For each τ, solve government budget constraint for r and evaluate FOC

The missing RSS

- Assume "correct" social discount factor, $\delta = \beta$. Left hand side of FOC:

liquidity **benefit** of greater debt

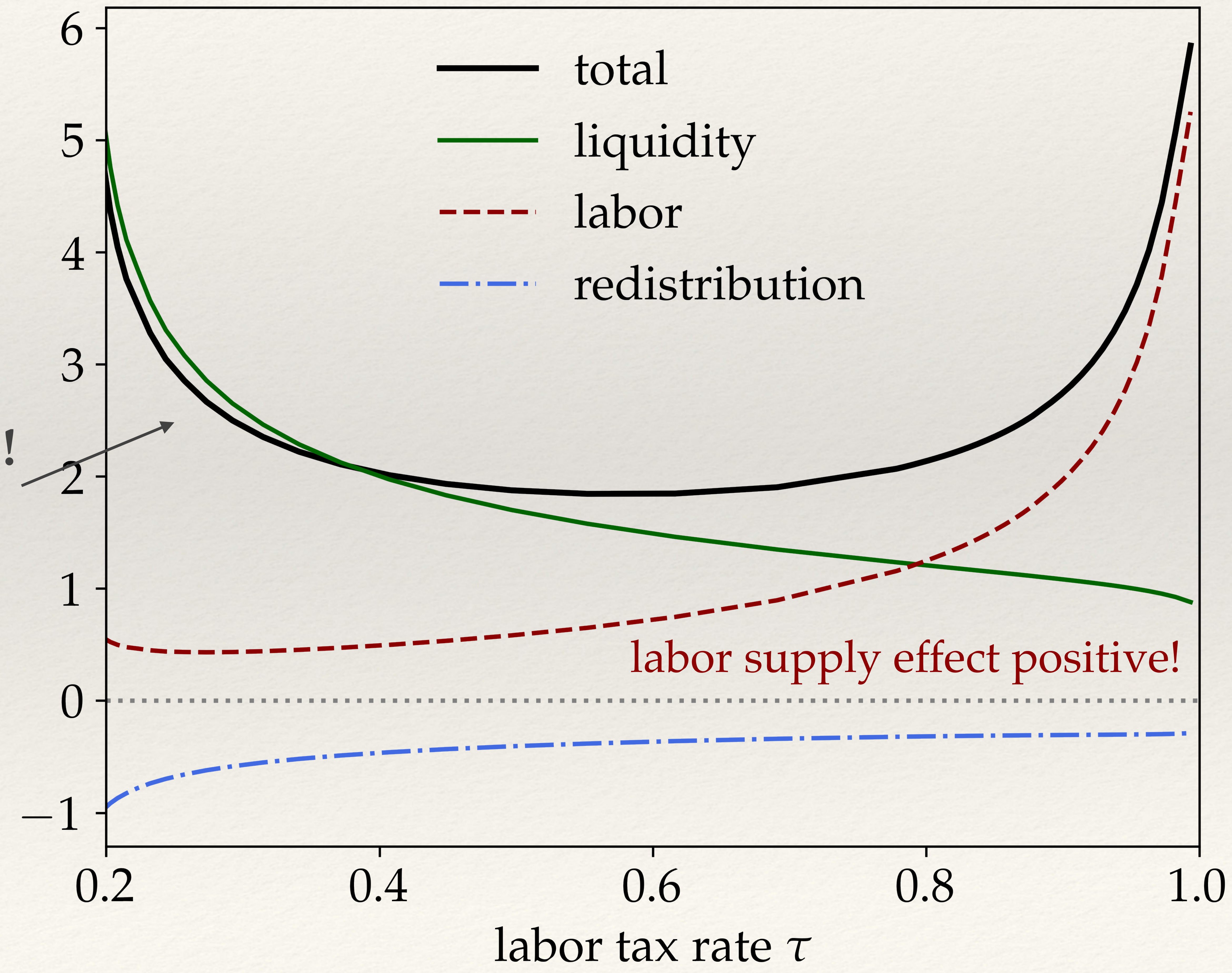
$$\left(1 - \beta\left(1 + r\right)\right) \ell \left(m\epsilon^{A,r} + \epsilon^{A,\tau}\right)$$

$$-\frac{\tau}{1-\tau}\left(-\epsilon^{N,\tau} - m\epsilon^{N,r}\right) - \left(\ell\, m - 1\right)$$

cost: redistribution

benefit: greater labor supply

Always > 0 !
No RSS!



Optimal steady state exists

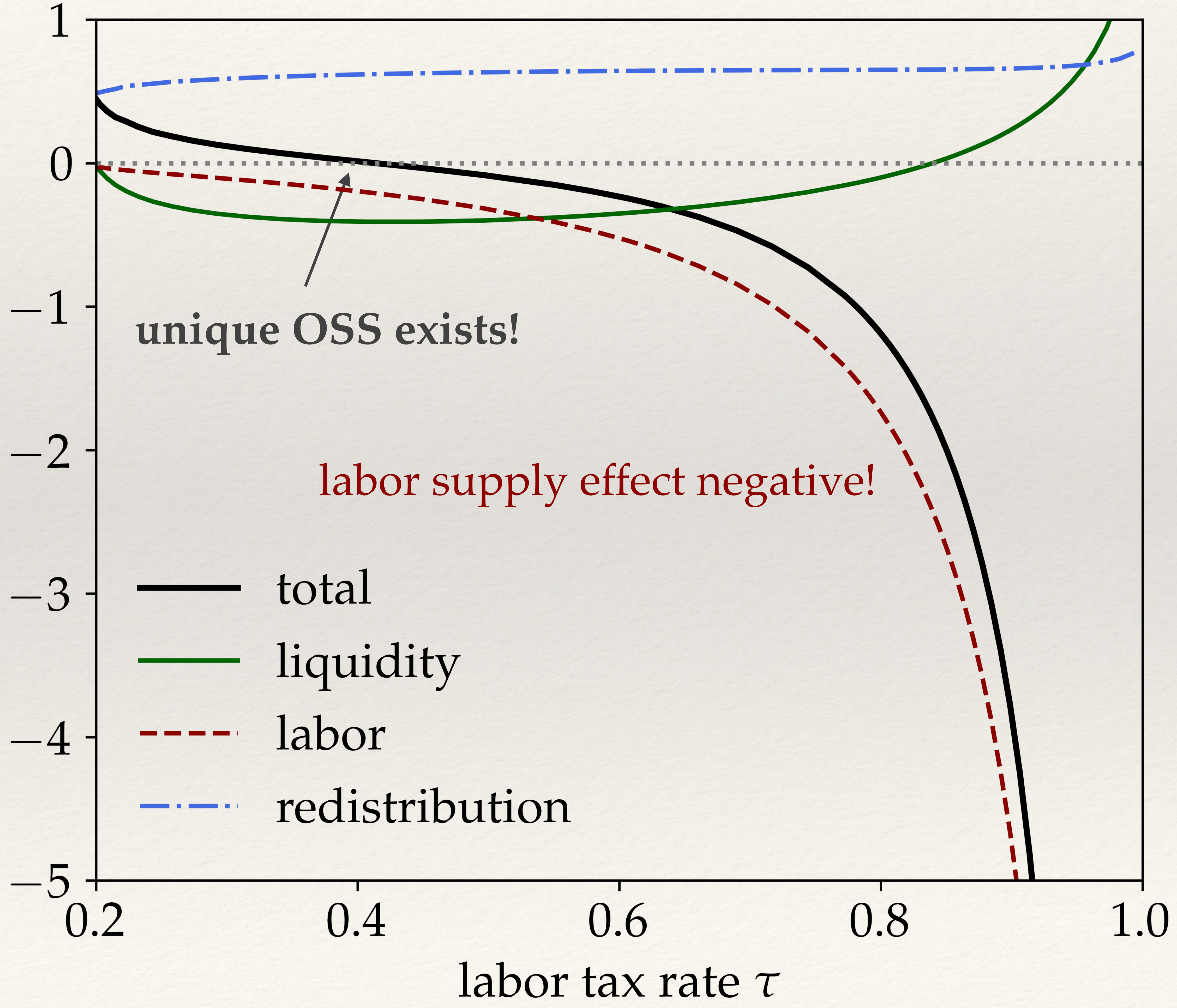
- Same with infinitely patient planner, $\delta = 1$:

liquidity **benefit** of greater debt

$$(1-(1+r))\,\ell\left(m\epsilon^{A,r}+\epsilon^{A,\tau}\right)$$

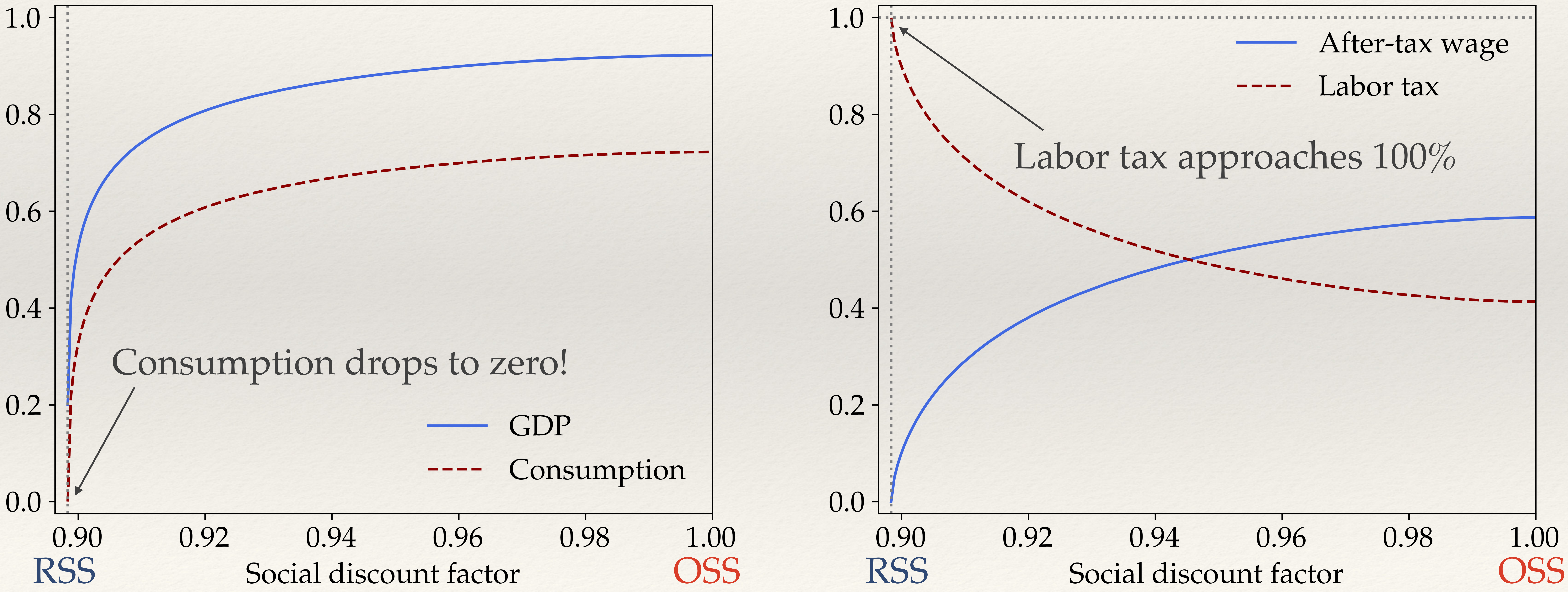
$$-\frac{\tau}{1-\tau}\left(-\epsilon^{N,\tau}-m\epsilon^{N,r}\right)-\left(\ell\, m-1\right)$$

cost: lower labor supply **cost**: redistribution



How the RSS vanishes

- Next, vary social discount factor δ between β and 1:



Standard Aiyagari economy: Why no RSS?

Benefits and **costs** to greater liquidity and higher labor taxes

liquidity benefit

labor supply ↑

~~labor supply ↓~~

redistribution

cost of redistribution is quantitatively small!

What if you change the parameterization?

- Very robust finding: $\tau_t \to$ **very close to immiseration!**
  - holds for any balanced growth preferences (King-Plosser-Rebelo)

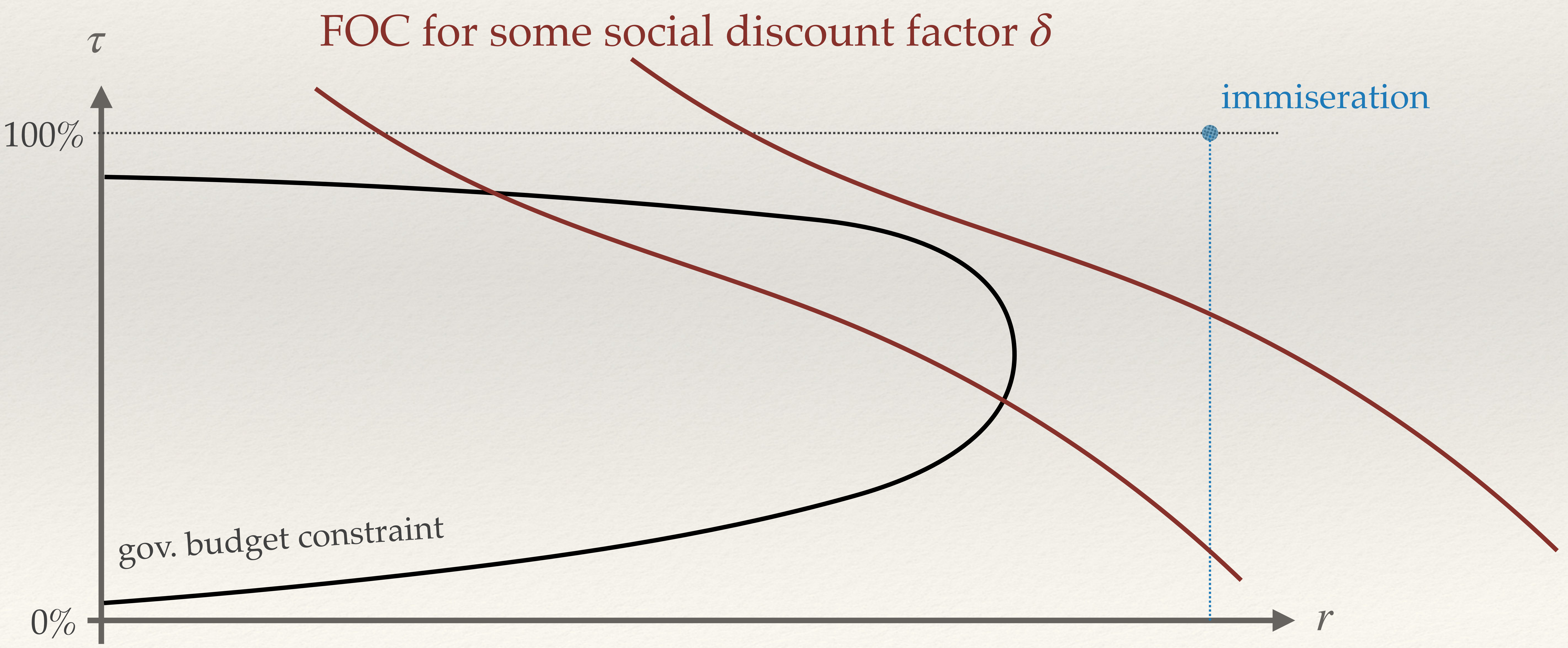
- Non-balanced growth preferences?
  - Strong income effects on labor supply: still find (near-) immiseration
  - Strong substitution effects (e.g. GHH): multiple interior RSS

GHH preferences?

$$u(c,n) = \frac{\left(c - \phi \frac{n^{1+\nu}}{1+\nu}\right)^{1-\sigma} - 1}{1-\sigma}$$

No wealth effect on labor supply

→ high taxes reduce labor supply & liquidity creation



Taking stock

- New method to compute Ramsey steady states in richer models than RA, TA
- Discounted elasticities of “sequence space” functions are key!
- **Insight:** RSS doesn’t exist for a standard balanced-growth Aiyagari model!
- Wide open field with many possible applications !