Information frictions

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Information frictions

- So far, have assumed full information & rational expectations ("FIRE")
- **Next:** Deviations from FIRE ("information frictions")
  - incomplete information (e.g. noisy information, sticky information)
  - deviations from rational expectations (e.g. cognitive discounting, level *k* thinking)
- Leading contender to explain **key puzzles** in macro & finance, e.g.
  - Why do $\{ \pi_t , I_t , C_t \}$ respond so **sluggishly** to aggregate shocks? (but not to idiosyncratic shocks)
  - Why do asset prices **overreact** to shocks?

A slight problem

- Deviations from FIRE already hard to simulate within simple RA models!
  - e.g. Mankiw Reis 2007, Mackowiak Wiederholt 2015
- **Goal:** Coherent framework to model *and simulate* deviations from FIRE
  - … not just RA, but also **HA !** (or any other block …)
- Materials here mostly a version of the approach we have developed for "Micro Jumps, Macro Humps…"

Introductory example

Monetary policy with myopic agents

- IKC equation for monetary policy

$$d\mathbf{Y} = \mathbf{M}^r d\mathbf{r}^{ante} + \mathbf{M} d\mathbf{Y}$$

$\mathbf{M}^r \equiv \frac{\partial \mathscr{C}}{\partial r}$ and $\mathbf{M} \equiv \frac{\partial \mathscr{C}}{\partial Y}$ are Jacobians of some household side (HA, RA, ZL…)

- Imagine households are **myopic:**
  - only start responding to dr_t^{ante} at date t
  - only start responding to dY_t at date t
- What is $d\mathbf{Y}$ in this case?

Manipulating Jacobians

- Take the intertemporal MPC matrix … **Is it still correct with myopia?**

Response to income increase at $s = 0$

$$\mathbf{M} = \begin{pmatrix} M_{00} & M_{01} & M_{02} & M_{03} & \cdots \\ M_{10} & M_{11} & M_{12} & M_{13} & \cdots \\ M_{20} & M_{21} & M_{22} & M_{23} & \cdots \\ M_{30} & M_{31} & M_{32} & M_{33} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Still correct with myopia? ✓

Manipulating Jacobians

- Take the intertemporal MPC matrix … **Is it still correct with myopia?**

Response to income increase at $s = 1$

$$\mathbf{M} = \begin{pmatrix} M_{00} & M_{01} & M_{02} & M_{03} & \cdots \\ M_{10} & M_{11} & M_{12} & M_{13} & \cdots \\ M_{20} & M_{21} & M_{22} & M_{23} & \cdots \\ M_{30} & M_{31} & M_{32} & M_{33} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Still correct with myopia? ✓ ✗

Manipulating Jacobians

- Take the intertemporal MPC matrix … **Is it still correct with myopia?**

Response to income increase at $s = 2$

$$\mathbf{M} = \begin{pmatrix} M_{00} & M_{01} & M_{02} & M_{03} & \cdots \\ M_{10} & M_{11} & M_{12} & M_{13} & \cdots \\ M_{20} & M_{21} & M_{22} & M_{23} & \cdots \\ M_{30} & M_{31} & M_{32} & M_{33} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Still correct with myopia? ✓ ✗ ✗

Manipulating Jacobians

- Take the intertemporal MPC matrix … **Is it still correct with myopia?**

Response to income increase at $s = 3$

$$\mathbf{M} = \begin{pmatrix} M_{00} & M_{01} & M_{02} & M_{03} & \cdots \\ M_{10} & M_{11} & M_{12} & M_{13} & \cdots \\ M_{20} & M_{21} & M_{22} & M_{23} & \cdots \\ M_{30} & M_{31} & M_{32} & M_{33} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Still correct with myopia? ✓ ✗ ✗ ✗

Do we need to modify the other entries in each column?

Manipulating Jacobians

- Take the intertemporal MPC matrix … **Is it still correct with myopia?**

$$\mathbf{M} = \begin{pmatrix} M_{00} & 0 & M_{02} & M_{03} & \cdots \\ M_{10} & M_{00} & M_{12} & M_{13} & \cdots \\ M_{20} & M_{10} & M_{22} & M_{23} & \cdots \\ M_{30} & M_{20} & M_{32} & M_{33} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Manipulating Jacobians

- Take the intertemporal MPC matrix … **Is it still correct with myopia?**

$$\mathbf{M} = \begin{pmatrix} M_{00} & 0 & 0 & M_{03} & \cdots \\ M_{10} & M_{00} & 0 & M_{13} & \cdots \\ M_{20} & M_{10} & M_{00} & M_{23} & \cdots \\ M_{30} & M_{20} & M_{10} & M_{33} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Manipulating Jacobians

- Take the intertemporal MPC matrix … **Is it still correct with myopia?**

$$\mathbf{M} = \begin{pmatrix} M_{00} & 0 & 0 & 0 & \cdots \\ M_{10} & M_{00} & 0 & 0 & \cdots \\ M_{20} & M_{10} & M_{00} & 0 & \cdots \\ M_{30} & M_{20} & M_{10} & M_{00} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

After date s, $M_{t,s}$ is just like the date $t-s$ response to an unanticipated shock!

Expectations matrix

- Another way to look at this: What are **expectations** about a date-s shock?
- Define matrix $\mathbf{E}$ that in column s has the expectations about date-s shock of 1

$$\mathbf{E} = \begin{pmatrix} 1 & 1 & 1 & 1 & \cdots \\ 1 & 1 & 1 & 1 & \cdots \\ 1 & 1 & 1 & 1 & \cdots \\ 1 & 1 & 1 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \qquad \rightsquigarrow \mathbf{E} = \begin{pmatrix} 1 & 0 & 0 & 0 & \cdots \\ 1 & 1 & 0 & 0 & \cdots \\ 1 & 1 & 1 & 0 & \cdots \\ 1 & 1 & 1 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

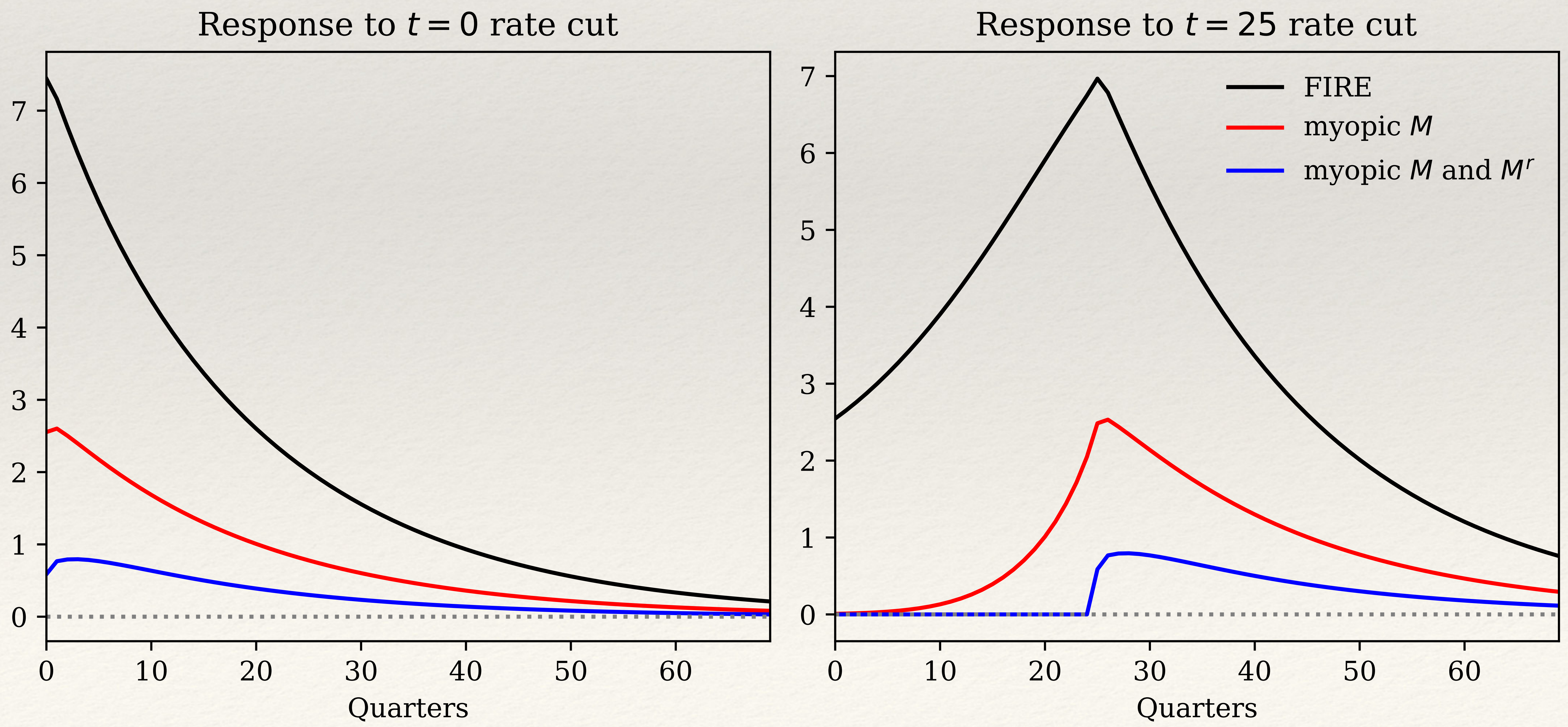
- $E_{t,s} dY_s$ is then the expected value of dY_s at date t.

Solving the myopic IKC

- How can we solve for the GE response of $d\mathbf{Y}$ then?

$$d\mathbf{Y} = \mathbf{M}^r d\mathbf{r}^{ante} + \mathbf{M} d\mathbf{Y}$$

- With zero new computational burden, we can solve our myopic economy!



Solving myopic IKC for fiscal policy

- Another application: Imagine we want to solve for fiscal multipliers but agents expect neither future taxes nor future income.
- What's the right IKC?

$$d\mathbf{Y} = d\mathbf{G} - \mathbf{M}d\mathbf{T} + \mathbf{M}d\mathbf{Y}$$

- **Next:** Generalize this to more general models of belief formation!

Two general assumptions we'll make

- We will make two implicit assumptions
- Agents are only "behavioral" about **future changes** in aggregate variables
  - steady state unaffected
  - not behavioral w.r.t. *idiosyncratic* income process
- Deviations from FIRE are **orthogonal** to idiosyncratic state
  - can relax, but too much today. See Guerreiro (2022).

Information frictions in the sequence space

General expectations matrix

- Consider a general $\mathbf{E} = (E_{t,s})$ matrix …
  - entry $E_{t,s}$ captures the date-t expectation of a unit shock at date s
  - $E_{t,s} dY_s$ is the date-t expectation of a shock dY_s at date s
- For today, make one of two assumptions (very common!)
  - agents have correct expectations about the shock by the time it hits
  - Jacobian $\mathbf{M}$ is such that knowledge of past shocks does not alter behavior

Typical example

$$\mathbf{E} = \begin{pmatrix} 1 & * & * & * & \cdots \\ 1 & 1 & * & * & \cdots \\ 1 & 1 & 1 & * & \cdots \\ 1 & 1 & 1 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Typical example

Like a news shock at date 1, that one period later dY goes up by 0.3

$$\mathbf{E} = \begin{pmatrix} 1 & 0.4 & \boxed{0.3} & 0.2 & \cdots \\ 1 & 1 & \boxed{0.6} & 0.4 & \cdots \\ 1 & 1 & 1 & 0.8 & \cdots \\ 1 & 1 & 1 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Typical example

Like a news shock at date 1, that two periods later dY goes up by 0.2

$$\mathbf{E} = \begin{pmatrix} 1 & 0.4 & 0.3 & \boxed{0.2} & \cdots \\ 1 & 1 & 0.6 & 0.4 & \cdots \\ 1 & 1 & 1 & 0.8 & \cdots \\ 1 & 1 & 1 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

General Jacobian manipulation

- Given **E** and FIRE Jacobian **M**, how do we come up with **M** ?
- Consider unit shock at date s. What is the response?
- At date τ, expectation shifts by $E_{\tau,s} - E_{\tau-1,s}$
  - **This is like a news shock with $s - \tau$ away!**

$$M_{t,s} = \sum_{\tau=0}^{\min\{t,s\}} \underbrace{\left(E_{\tau,s} - E_{\tau-1,s}\right) \cdot M_{t-\tau,s-\tau}}$$

date-t effect of a date-τ expectation revision about date-s shock

Here, convention: $E_{-1,s} = 0$

Examples

(1) Sticky information

- Mankiw Reis (2002) propose an information-based microfoundation of nominal rigidities
- Idea: a mass 1 of price setters would like to set their price equal to some markup over marginal cost

$$\log P_{it} = \log \mu + \log MC_t \qquad \text{where } MC_t \text{ is stochastic}$$

- **Only random fraction** $1 - \theta$ of price setters receive latest information in any given period. ($\theta = 0$ is flexible prices)
- This is called "**sticky information**".

(1) Nesting sticky information

- What is the Jacobian of $\log P_t$ to $\log MC_t$ in the model?
- With FIRE: $\mathbf{M} = \mathbf{I}$

$$\mathbf{E} = \begin{pmatrix} 1-\theta & 1-\theta & 1-\theta & \cdots \\ 1-\theta^2 & 1-\theta^2 & 1-\theta^2 & \cdots \\ 1-\theta^3 & 1-\theta^3 & 1-\theta^3 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \Rightarrow \mathbf{M} = \begin{pmatrix} 1-\theta & 0 & 0 & \cdots \\ 0 & 1-\theta^2 & 0 & \cdots \\ 0 & 0 & 1-\theta^3 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

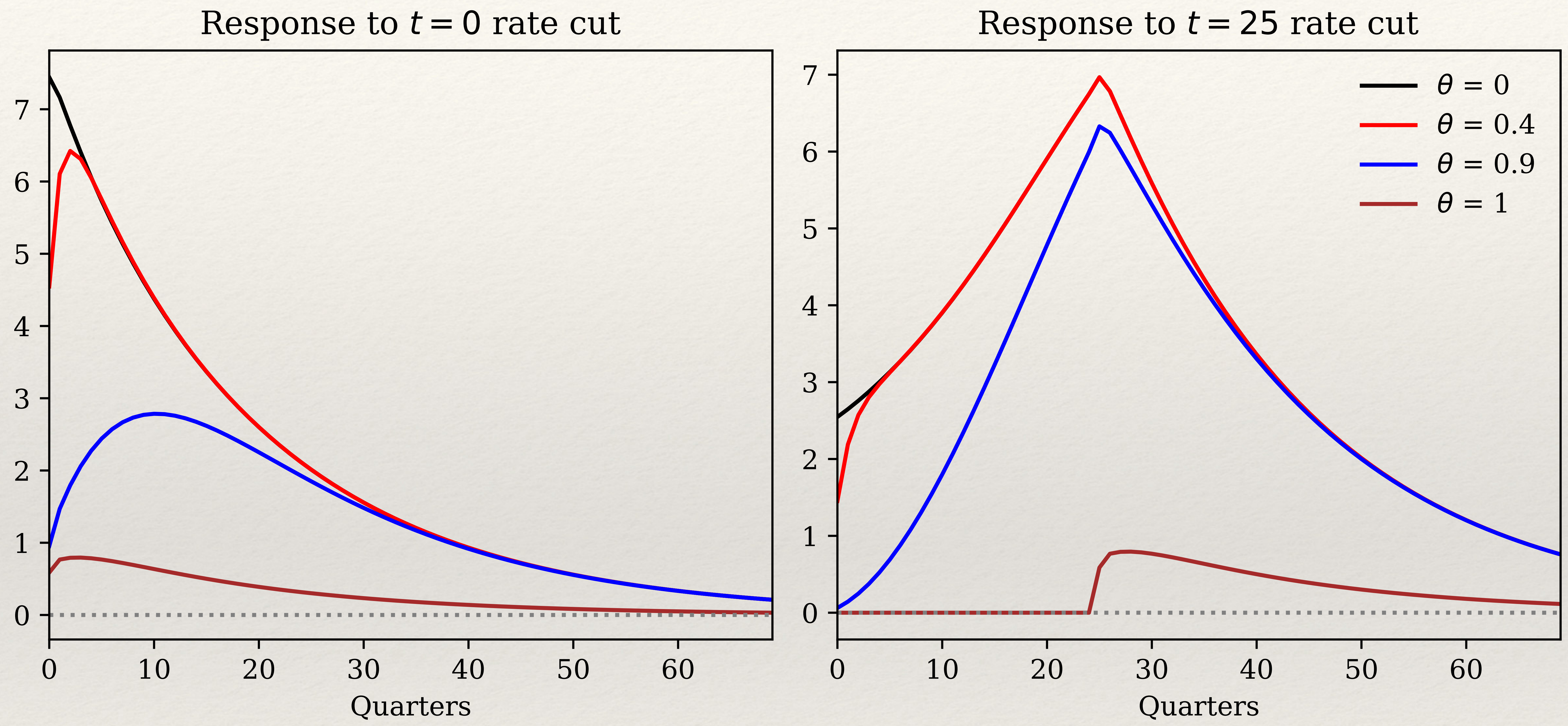
(2) Sticky expectations

- Sticky info works well if past shocks don't influence behavior
  - Not true for HA models!
- Carroll et al (2020) workaround: Assume everyone learns when shock hits!

$$\mathbf{E} = \begin{pmatrix} 1 & 1-\theta & 1-\theta & \cdots \\ 1 & 1 & 1-\theta^2 & \cdots \\ 1 & 1 & 1 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$\mathbf{M} = \begin{pmatrix} M_{00} & (1-\theta)M_{01} & (1-\theta)M_{02} & \cdots \\ M_{10} & (1-\theta)M_{11}+\theta M_{00} & (1-\theta)M_{12}+\theta(1-\theta)M_{01} & \cdots \\ M_{20} & (1-\theta)M_{21}+\theta M_{10} & \vdots & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

(2) HANK with sticky expectations



- Intermediate θ generates strong hump shape
- Nice way to replace habit and other slow-adjustment frictions in DSGE models

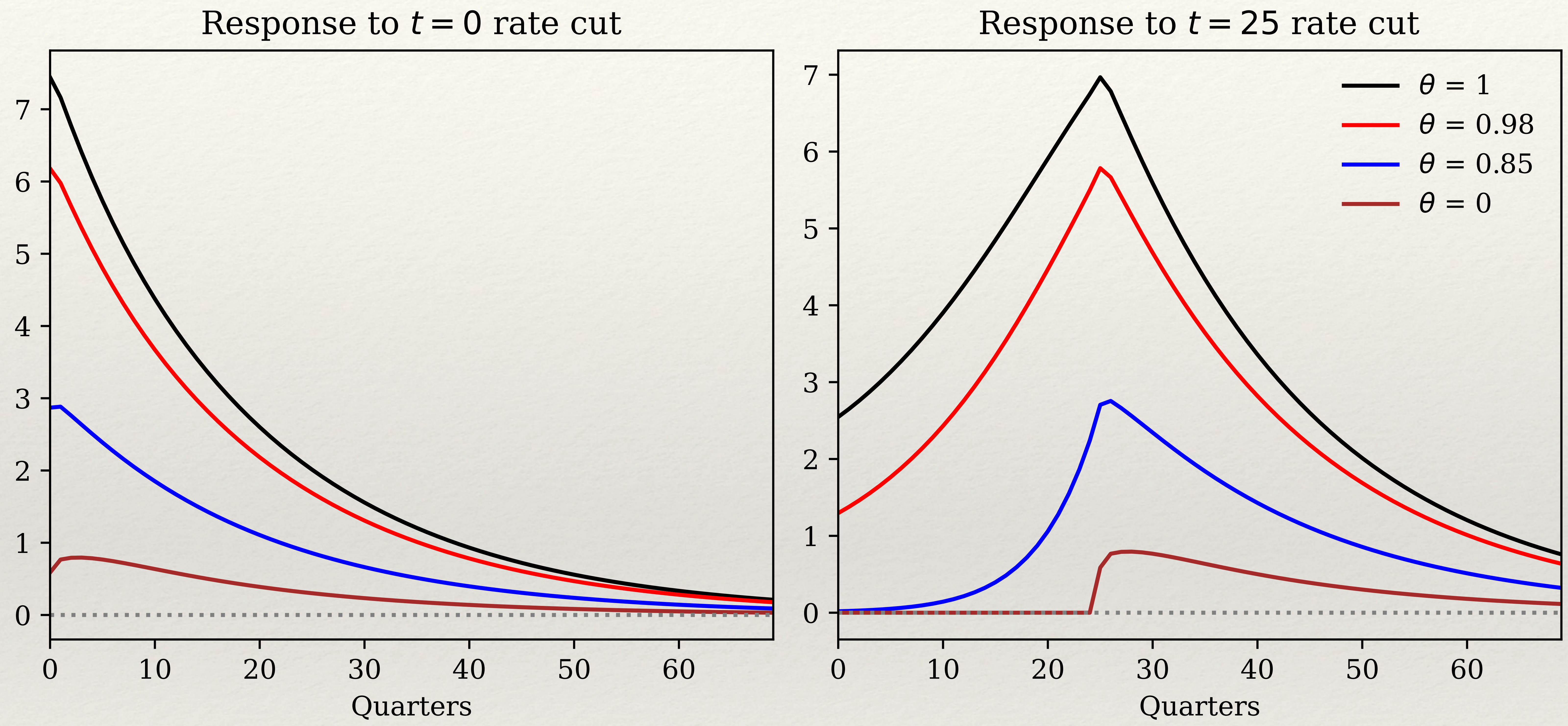
(3) Cognitive discounting

- Gabaix (2020) introduces **cognitive discounting**
- Idea: Agents respond to shock in h periods as if shock size is dampened by θ^h
  - this is as if agents *expect* shock size θ^h, instead of 1

$$\mathbf{E} = \begin{pmatrix} 1 & \theta & \theta^2 & \theta^3 & \cdots \\ 1 & 1 & \theta & \theta^2 & \cdots \\ 1 & 1 & 1 & \theta & \cdots \\ 1 & 1 & 1 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

- Here, dampening relative to *diagonal*
  - $\neq$ sticky info, where dampening relative to *initial period*

(3) HANK with cognitive discounting



- Doesn't generate humps so well, but dampens forward guidance!

(4) Level-k thinking

- Farhi Werning (2019) is first paper to combine HANK with deviations from FIRE
- They use **level-*k* thinking:**
  - $k = 1$: all agents believe output is at steady state
  - $k = 2$: all agents believe *all other* agents are at level $k = 1$
  - $k = 3$: all agents believe *all other* agents are at level $k = 2$, … etc

(4) Level-1 thinking

- Level $k = 1$ very close to our myopic example:

$$\mathbf{M}^{(1)} = \begin{pmatrix} M_{00} & 0 & 0 & 0 & \cdots \\ M_{10} & M_{00} & 0 & 0 & \cdots \\ M_{20} & M_{10} & M_{00} & 0 & \cdots \\ M_{30} & M_{20} & M_{10} & M_{00} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$d\mathbf{Y}^{(1)} = \mathbf{M}^r d\mathbf{r} + \mathbf{M}^{(1)} \cdot d\mathbf{Y}^{(1)}$$

(4) Level-2 thinking

- What about level-2?

$$d\mathbf{Y}^{(2)} = \mathbf{M}^r d\mathbf{r} + \mathbf{M} \cdot d\mathbf{Y}^{(1)}$$

Everybody expects everyone else to spend money according to level-1! Hence everyone expects income = $d\mathbf{Y}^{(1)}$

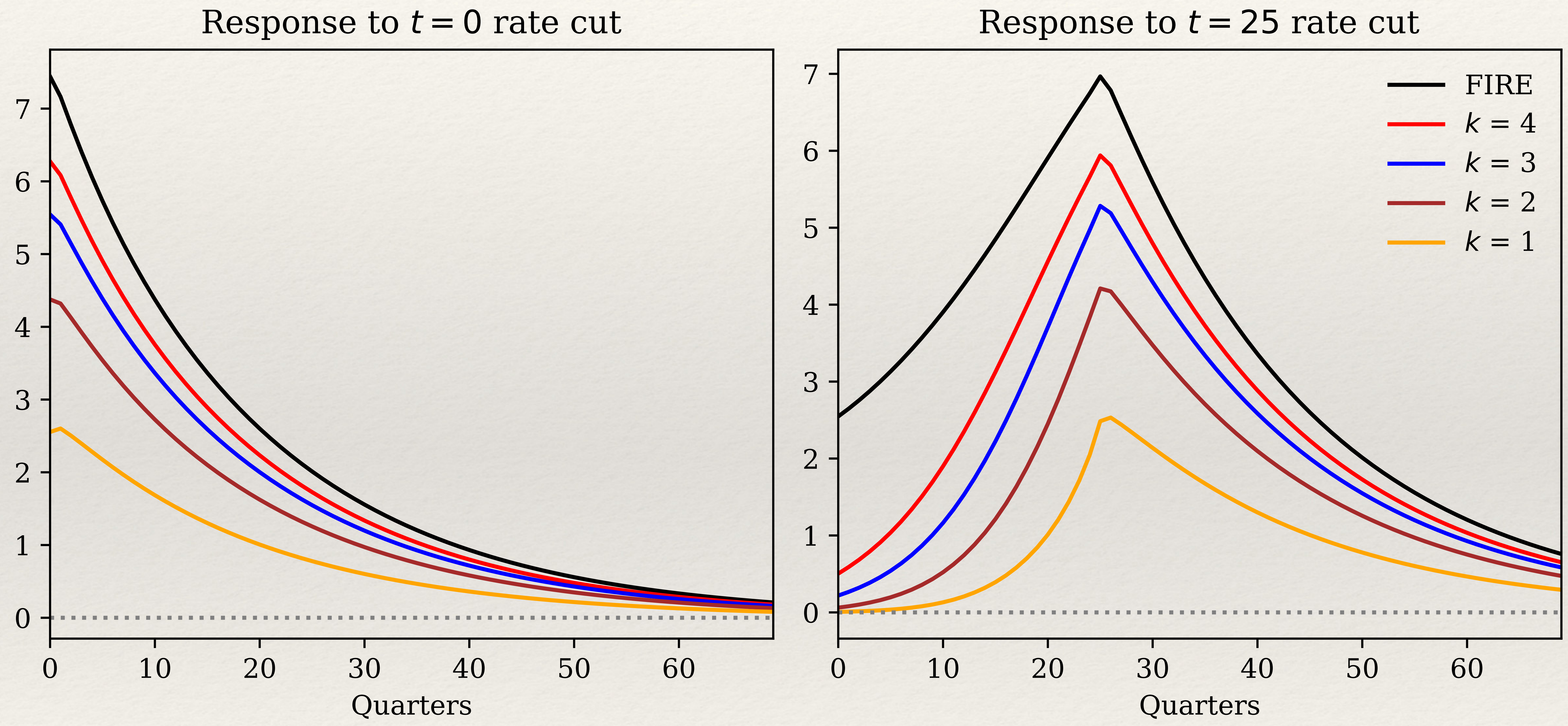
$$+\mathbf{M}^{(1)} \cdot \left(d\mathbf{Y}^{(2)} - d\mathbf{Y}^{(1)}\right)$$

… but actual income is $d\mathbf{Y}^{(2)}$!

Agents are constantly surprised when actual income $d\mathbf{Y}^{(2)}$ differs from $d\mathbf{Y}^{(1)}$

General recursion: $d\mathbf{Y}^{(k+1)} = \mathbf{M}^r d\mathbf{r} + \mathbf{M} \cdot d\mathbf{Y}^{(k)} + \mathbf{M}^{(1)} \cdot \left(d\mathbf{Y}^{(k+1)} - d\mathbf{Y}^{(k)}\right)$

(4) HANK with level k



Takeaway

Conclusion

- Information rigidities can be nested quite nicely in the sequence space
- Not just gives us a straightforward way of simulating them for RA models,
  - but allows us to apply it to HA models equally well!