Lecture 4
Monetary policy

Adrien Auclert

Goethe Heterogeneous-Agent Macro Workshop
June 2024

Class plan

Yesterday: The canonical HANK model & fiscal policy

Today: Monetary policy

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Today: Monetary policy

We focus mostly on real interest rate rules in closed economies

At the end, we'll study Taylor rules.

See 2022 NBER workshop for open economy.

Roadmap

1. Review of monetary policy in the standard NK model
2. Monetary policy in the canonical HANK model
3. Direct and indirect effects of monetary policy
4. Cyclical income risk
5. Takeaway

Review of monetary policy in the standard NK model

- Recall the standard 3-equation NK model
- Separable preferences, sticky prices or wages, perfect foresight
- Standard derivation gives

$$c_t = c_{t+1} - \sigma^{-1}(i_t - \pi_{t+1}) \tag{EE}$$

$$\pi_t = \kappa c_t + \beta \pi_{t+1} \tag{NKPC}$$

$$i_t = \pi_{t+1} + \epsilon_t \tag{r-rule}$$

- [NB: with Taylor rule, $i_t = \phi \pi_t + \epsilon_t$ replaces (r-rule), usually with $\phi > 1$]

Monetary propagation in the NK model

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1. expansion in c_t so output $y_t \uparrow$, inflation $\pi_t \uparrow$
2. far out shocks to ϵ_t with large t are not dampened ("forward guidance puzzle")

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Two key issues with this model:

- **transmission into consumption:** 100% via Euler equation (implausible?)
- **output response:** forward guidance puzzle, model too forward looking

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- McKay et al. (2016): borrowing constraints make consumption less forward looking. Want to get something like

$$c_t = \delta c_{t+1} - \sigma^{-1}(i_t - \pi_{t+1}) \qquad \text{with } \delta < 1 \tag{DEE}$$

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Next: What HANK actually does!

Monetary policy in the canonical HANK model

Setting up the model

- Take canonical HANK model, but abstract from fiscal policy
  - $T = \tau = G = B = 0$
  - but allow agents to borrow from each other: $\underline{a} < 0$ (as in Huggett model)
  - next lecture, bring back government to study monetary-fiscal interactions
- Real rate rule: monetary policy sets $r_t^{ante} = i_t - \pi_{t+1}$ directly

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  1. Output response relative to RA? (Magnitude? Any "discounting"?)
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We'll start with 1.

Back to our equilibrium conditions

Under these assumptions, the canonical HANK model can just be written as:

$$\max_{c_{it}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(u(c_{it}) - v\left(N_t\right)\right)$$

$$c_{it} + a_{it} \leq (1 + r_{t-1}^{ante})a_{it-1} + e_{it} Y_t$$

$$a_{it} \geq \underline{a}$$

with

$$C_t \equiv \int c_{it} di = Y_t = N_t$$

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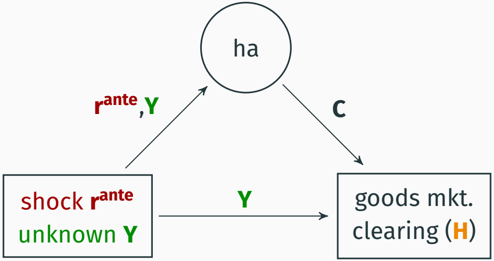
That's it!

Let's visualize this as a DAG:



DAG of this model

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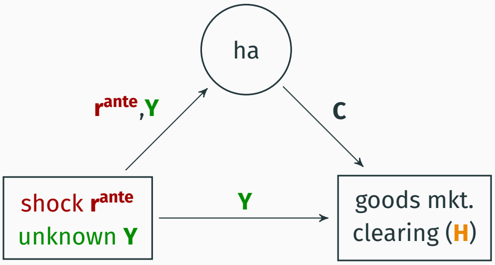

Here again, simple fixed point:

$$\mathcal{C}_t\left(\{r_s^{ante}, Y_s\}\right) = Y_t$$

- In practice, we usually write `HetBlocks` with "ex-post r" convention, i.e. here:

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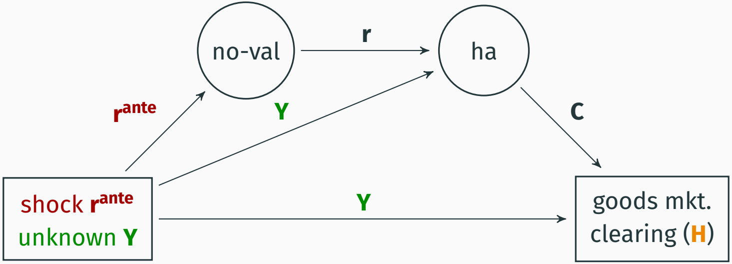
- This is more general: allows us to handle valuation effects (see next lecture)
- Here there are no valuation effects, so we just have

$$r_t = r_{t-1}^{ante} \quad t \geq 1$$

$$r_0 = r_{ss}$$

- This adds one "no valuation" block to the DAG

Our new DAG is:



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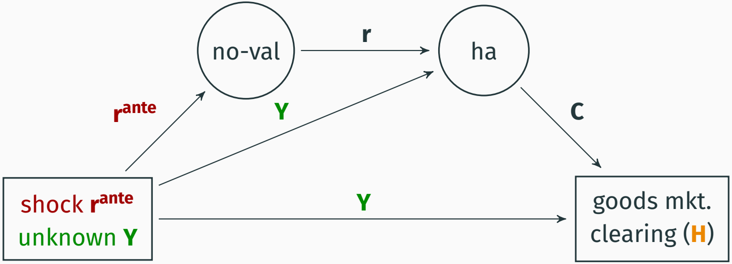

We can use `CombinedBlock` in SSJ to do the convolution

$$\tilde{\mathcal{C}}_t\left(\{r_s^{ante}, Y_s\}\right) \equiv \mathcal{C}_t\left(\{r_j\left(r_s^{ante}\right), Y_s\}\right)$$

This way, we are back to our simple fixed point:

$$\tilde{\mathcal{C}}_t\left(\{r_s^{ante}, Y_s\}\right) = Y_t$$

- As in fiscal lecture, let's linearize this sequence space equation
- Define $d\mathbf{r}^{ante} \equiv \left(dr_0^{ante}, dr_1^{ante}, \ldots\right)$, and let $d\mathbf{Y} = (dY_0, dY_1, \ldots)$ as before. Define Jacobian $\mathbf{M}^r \equiv \left(\partial\tilde{\mathcal{C}}_t/\partial r_s^{ante}\right)_{t,s}$ capturing direct effect of r on C.

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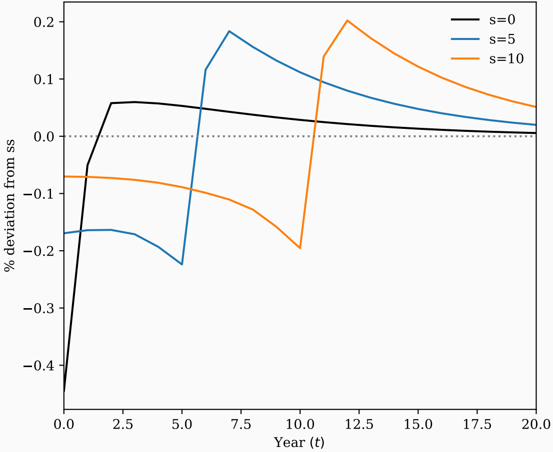
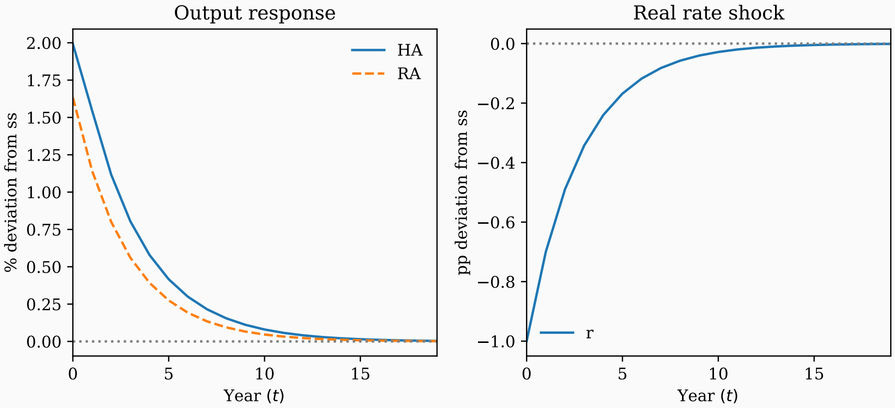
Next: Let's visualize $\mathbf{M}^r$; then the solution $d\mathbf{Y}$ for an AR(1) shock to $d\mathbf{r}^{ante}$

Columns of Jacobian $\mathbf{M}^r$



Monetary policy shock in HA (AR(1) with $\rho = 0.7$)



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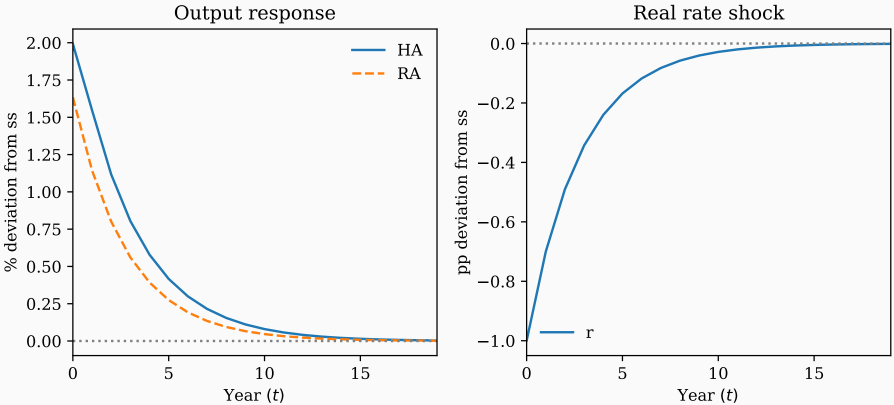

- HA $>$ RA! Interesting! But why?

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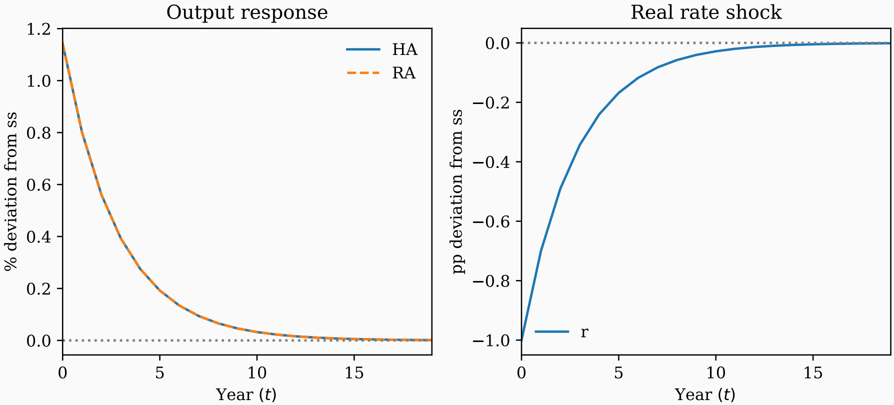
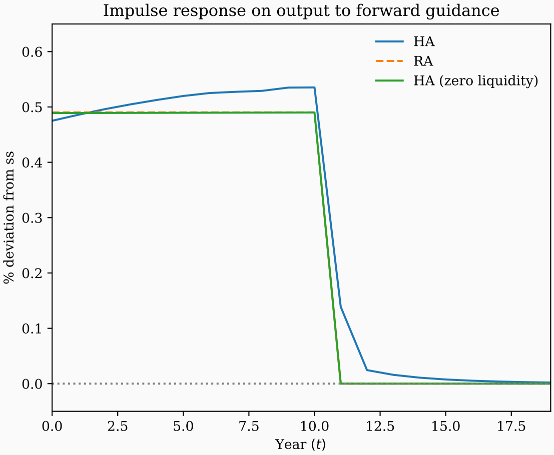
  $\rightarrow$ Werning (2015)'s **neutrality result** for zero liquidity and acyclical income risk
- In particular: No discounting in log-linearized Euler equation!

Neutrality for monetary policy in the ZL limit



Neutrality also implies the forward guidance puzzle is not solved by HA



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- No robust result that HA $\neq$ RA !
  - in fact, with zero liquidity, we showed that HA $=$ RA!
  - forward guidance can be equally powerful
- But how can that be, given that HA breaks the Euler equation?
- **Next: study transmission channels**

Direct and indirect effects of monetary policy

- To see what's going on, let's go back to our IKC-like equation:

$$d\mathbf{Y} = d\mathbf{C} = \underbrace{\mathbf{M}^r \cdot d\mathbf{r}^{ante}}_{\text{Direct effect}} + \underbrace{\mathbf{M} \cdot d\mathbf{Y}}_{\text{Indirect effect}}$$

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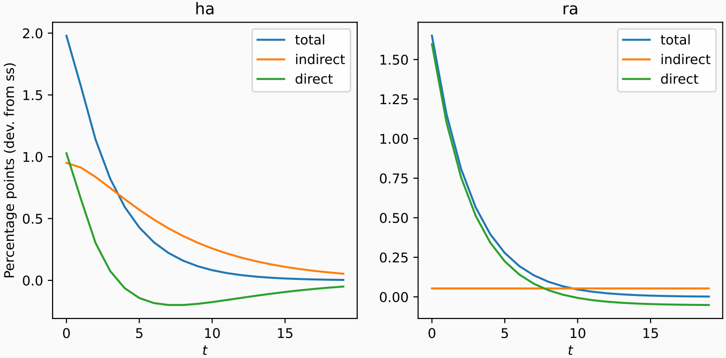
- Why? High MPCs make C more sensitive to Y but also **less sensitive to** r^{ante}!
  - cf Auclert (2019): substitution effect of dr^{ante} scales with $-\sigma^{-1}(1 - MPC)$
  - In ZL model, can actually prove that $\mathbf{M}^r = -\sigma^{-1}(\mathbf{I} - \mathbf{M})\mathbf{U}$ so

$$d\mathbf{C} = -\sigma^{-1}(\mathbf{I} - \mathbf{M})\,\mathbf{U} \cdot d\mathbf{r}^{ante} + \mathbf{M} \cdot d\mathbf{Y}$$

Decomposition into direct and indirect effects

- Let's implement $d\mathbf{C} = \mathbf{M}^r d\mathbf{r}^{ante} + \mathbf{M} \cdot d\mathbf{Y}$ in our canonical HA model:



- This is the key result from Kaplan et al. (2018); has proved very robust

Cyclical income risk

Introducing cyclical income risk

- A simple way to introduce cyclical income risk by adopting different labor allocation rule. Auclert and Rognlie (2018) propose

$$n_{it} = Y_t \frac{(e_{it})^{\zeta \log Y_t}}{\mathbb{E}\left[e_i^{1+\zeta \log Y_t}\right]} \equiv Y_t \Gamma\left(e_{it}, Y_t\right)$$

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- Distribution of income $y_{it} \equiv e_{it} n_{it}$ now reacts to monetary policy

$$\text{sd}\left(\log y_{it}\right) = \left(1 + \zeta \log Y_t\right) \text{sd}\left(\log e_i\right)$$

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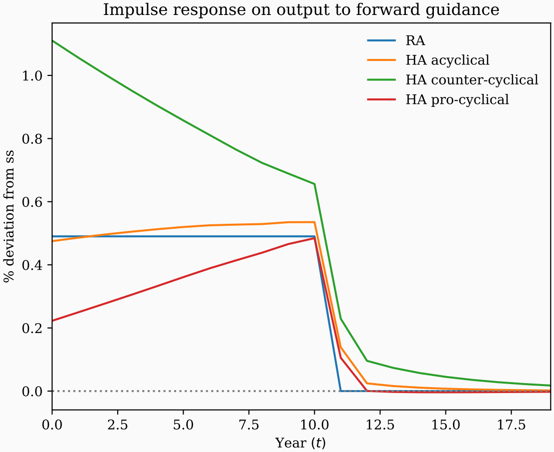
- Matters because:
  - current shocks redistribute between different MPCs ("cyclical inequality")
  - future shocks change income risk ("cyclical risk")

Countercyclical income risk makes the forward guidance puzzle worse!

- Consider a r_T shock with three calibrations for ζ in HA model



Zero liquidity limit with cyclical income risk

▸ details

What's going on? In ZL limit, we get an **exact** discounted Euler equation

$$y_t = \delta \cdot \mathbb{E}_t \left[y_{t+1} \right] - \sigma^{-1} \cdot \text{const} \cdot \left(r_t^{ante} - \log \left(\beta \overline{\rho} \right) \right)$$

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1. Dynamic discounting ($\delta < 1$) $\Leftrightarrow \zeta > 0$ procyclical risk (less common)

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What's going on? In ZL limit, we get an **exact** discounted Euler equation

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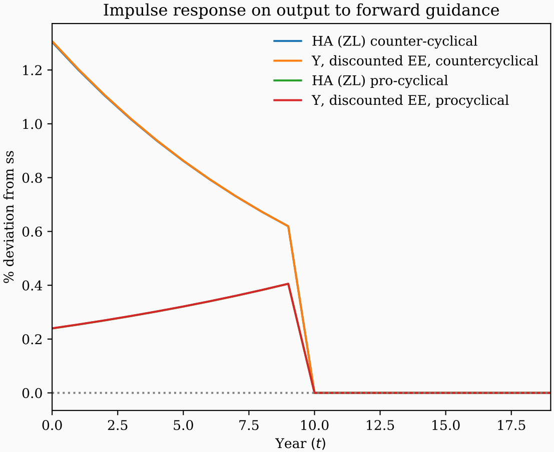
3. Dynamic neutrality ($\delta = 1$) $\Leftrightarrow \zeta = 0$ acyclical risk, as in Werning

Why? Precautionary savings. Think about logic of discounted Euler equation.

- In the empirically plausible case, the fwd guidance puzzle is **aggravated**!
  Acharya and Dogra (2020), Bilbiie (2024)



Indirect ways to make income risk cyclical

- In richer models income of agents typically involves multiple components,

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  $\Rightarrow$ both τ_t and T_t fall after expansionary r_t^{ante} (why?)
- If τ_t allocated to highest income state and T_t to all $\Rightarrow$ procyclical risk!
- These are the assumptions in McKay et al. (2016).
  - Reason why that paper "solves" the forward guidance puzzle!

Summary: Cyclical income risk

- Cyclical income risk matters
- Procyclical income risk $\Rightarrow$ weakens monetary policy + fwd guidance
  - … but not empirically supported
- Countercyclical income risk is empirically more plausible
  - … but aggravates forward guidance puzzle!

Takeaway

1. HA model does not imply robustly different output response
    - Except to the extent that income risk is pro/countercyclical
2. But it *does* change transmission: indirect effects are more important!

Takeaway: Monetary policy with heterogeneous agents

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1. HA model does not imply robustly different output response
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2. But it *does* change transmission: indirect effects are more important!
    - This is the main result in KMV. Why do we care about that per se?
    - KMV: labor & financial market institutions matter more than we thought
    - We'll see other reasons for why we should care in the next lecture

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Zero liquidity limit with cyclical income risk

back

- Take ZL model with cyclical income risk. Euler for $\overline{s}$:

$$\left(Y_t \Gamma\left(\overline{e}, Y_t\right)\right)^{-\sigma} = \beta\left(1 + r_t^{ante}\right) \mathbb{E}_t\left[\left(Y_{t+1} \Gamma\left(e', Y_{t+1}\right)\right)^{-\sigma} | \overline{e}\right]$$

- Log-linearize around steady state $\Rightarrow$

$$y_t = \delta \mathbb{E}_t\left[y_{t+1}\right] - \sigma^{-1} \gamma(\overline{e})^{-1}\left(r_t^{ante} - \log\left(\beta \overline{\rho}\right)\right)$$

where, if $\gamma(e) \equiv 1 + \frac{\Gamma_Y Y}{\Gamma}$ is the elasticity of income wrt Y for agent in s:

$$\delta \equiv \overline{\rho}^{-1} \mathbb{E}\left[(e/\overline{e})^{-\sigma} \frac{\gamma(e)}{\gamma(\overline{e})} | \overline{e}\right] = \sum \omega(e) \frac{\gamma(e)}{\gamma(\overline{e})} \qquad \text{where } \sum_e \omega(e) = 1$$

- What matters is cyclicality of $y(\overline{e})$ relative to other income states
- Example with two states: $\delta = 1 - \omega + \omega \frac{\gamma_L}{\gamma_H}$ with $\omega \in (0, 1)$